Kinetic theory of electrons and positrons in dense gaseous and liquid systems

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The recent resurgence of interest in positron transport in gases has been driven both by new fundamental positron-atom/molecule cross scattering sections [1, 2], and by the richness and novelty of the associated transport phenomena [3, 4]. In particular the phenomenon of negative differential conductivity (NDC) induced by positronium formation continues to be a focus of attention of kinetic theorists and modellers [5, 6], and there are other interesting effects yet to be explained. Although this work in gaseous systems is largely motivated by intrinsic physical interest, there are a number of important technological and medical applications (e.g. PET scans) which provide an additional imperative for such studies and in particular their extension to dense systems (liquid and soft-condensed), the subject of this presentation. Investigations of positron transport in dense systems so far have been very limited [7]. One can draw to some extent on the extensive transport theory literature for electrons in dense gases and liquids [8], and both electrons and positrons in gases, but there is no straightforward way of directly adapting this existing transport theory: A new theory is needed, in which the effects of both non-reactive coherent scattering by many atoms in the dense gaseous and liquid phases, and reactive collisions are accounted for, through a dynamic structure factor \(S(K,\Omega)\) and the positronium formation at cross section respectively.

In a nutshell, the broad aim is to outline a general kinetic theoretical framework for calculation of positron transport coefficients in dense system gaseous systems and liquids. The procedure is as follows: Firstly, we write down a kinetic equation for a structure medium, incorporating both coherent elastic scattering, exitation, ionization and positronium formation effects; and secondly, the experience gained over the past thirty years or so in gaseous electronics is brought to bear on solving this kinetic equation, in particular:

1. With regard to the quantities to be calculated from the kinetic equation for purposes of comparison with experiment, it has been established for transport in gaseous media that reactive collisions generally produce two distinct families of transport properties, so-called “bulk” and “flux” coefficients, defined via the diffusion equation and the flux-gradient equation (Fick’s law) respectively respectively. Generally speaking, the former are measurable and the latter are not, and the differences can be substantial, both qualitatively and quantitatively speaking. A similar duality carries over to dense gaseous and liquid phases, and reactive collisions are accounted for, through a dynamic structure factor \(S(K,\Omega)\) and the positronium formation at cross section respectively.

2. With regard to the solution of the kinetic equation, it is now firmly established in the literature [9] that any assumption of near-isotropy of the charged particle (electron or positron) distribution function \(f(c)\) with respect to velocities \(c\) may lead to inaccuracies in transport coefficients. In particular, one should avoid the limitations imposed by the “two-term” spherical harmonic \(Y_m^n(c)\) representation of \(f(c)\) and seek a “multiterm” (arbitrary number of spherical harmonics) solution of the kinetic equation. In this regard, it is important to note that in general two angles \(\hat{c} = (\theta, \phi)\) and an expansion in spherical harmonics are required to specify \(f(c) = f(c, \theta, \phi)\) in velocity space. Representation of \(f(c)\) in terms of one angle via and expansion Legendre polynomials \(P_l(\theta)\) is valid if and only if the problem at hand is axially symmetric.
We begin with a benchmark system considering the non-equilibrium transport of electrons in a gas of hard spheres. The structure of the medium in this case can be described by the well known analytic Percus-Yevick relation and is a function of the volume fraction of particles to space $\phi$ for a given physical cross-section. There are a number of interesting properties that emerge in the transport coefficients from the inclusion of coherent scattering from a structured medium. These include the phenomena of structure-induced negative differential conductivity and structure-induced anisotropic diffusion. That latter is displayed in Figure 1.

![Impact of the volume fraction $\phi$ on the anisotropic nature of diffusion for electrons in a gas of hard spheres.](image)

**Fig. 1:** Impact of the volume fraction $\phi$ on the anisotropic nature of diffusion for electrons in a gas of hard spheres.

Also, we will consider the non-equilibrium transport of positrons in liquid argon not only because this is of interest in its own right, but also because the results are very different from the much-studied case of electrons in liquid argon [8]. It is also interesting to compare with recent results reported for positrons in dilute gaseous argon [5], and in particular to see the way in which liquid structure influences the onset of Ps-induced negative differential conductivity and the anisotropic nature of diffusion.

**References**